

Pion structure function in nuclear medium

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Abstract

We study the pion structure function in nuclear medium using the Nambu and Jona-Lasinio model, and its implication for the nuclear pion enhancement of the sea quark distribution in nuclei. By using the operator product expansion, medium effect of the nuclear matter is incorporated in calculations of the twist-2 operators. We find density dependence of the pion structure function is rather weak around the nuclear matter density. We also discuss how the medium modification of the pion structure affects the sea quark enhancement in the nucleus.

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1 Introduction

Modification of the hadron structure in nuclear medium is an interesting topic of the nuclear physics, and has attracted considerable interests. In the deep inelastic scattering, such modifications of the nucleon structure in nuclei are observed experimentally, that is, EMC effect[1]. It is very difficult to calculate the hadron structure function itself by using QCD directly, though the measured momentum dependence of the structure function is consistent with predictions of the perturbative QCD. Recently, several attempts were done to calculate the structure function in terms of low energy effective theories of QCD, which reproduce hadron properties such as masses and magnetic moments[2]. In those studies, it is assumed that calculation of the twist-2 matrix elements within the effective theory gives structure function at certain low energy scale $\mu \sim 1\text{GeV}$, where the effective theories make sense. The resulting structure function is evolved to the experimental high momentum scale with the help of the perturbative QCD, and comparison with experiments can be made[3, 4].

In particular, the pion structure function is of special interest so as to study the nuclear pion enhancement of the sea quark distribution in the deep inelastic scattering[5, 6] and the Drell-Yan (DY) process[7]. The nuclear pion enhancement due to the attractive nucleon-nucleon interaction in the nucleus was extensively studied in the last decade[8]. The nucleon-hole and delta-hole excitations shown in Fig.1 modify the pion-like mode in the nucleus, and lead an excess of the pion number in nuclei. Llewellyn Smith[5], and Ericson and Thomas[6] discussed that such an enhancement of the pion number changes the sea quark distribution in the nucleus. Assuming that a part of the nucleon sea quark distribution is originated from the virtual pion cloud, the sea quark distribution of the nucleus is written by the convolution formula[9];

$$\bar{q}_{\pi NN}^A(x, Q^2) = \int_x^1 dy/y f_{\pi NN}^A(y) \bar{q}_\pi(x/y, Q^2) \quad , \quad (1)$$

where $\bar{q}_{\pi NN}^A(x, Q^2)$ is the sea quark distribution of the nucleon in the nucleus originated from the pion cloud, and $\bar{q}_\pi(x_\pi, Q^2)$ the antiquark distribution in the pion. Following the work of Ericson and Thomas, the longitudinal pion momentum distri-

bution in the nucleus $f_{\pi NN}^A(y)$ is given by[6],

$$f_{\pi NN}^A(y) = 3 y \frac{g_{\pi NN}^2}{16\pi^2} \int_{m_N^2 y}^{\infty} dq \int_0^{q-m_N y} d\omega \frac{q^2}{(t+m_\pi^2)^2} [F_{\pi NN}(q^2)]^2 R(\omega, q) \quad , \quad (2)$$

where $g_{\pi NN}$ is the pion nucleon coupling constant, $F_{\pi NN}(t)$ the pion-nucleon form-factor, and m_N , m_π the masses of the nucleon and pion, respectively. Here, $R(\omega, q)$ is the response function of the nuclear matter calculated by the random phase approximation, and carries the information of the collective excitation illustrated in Fig.1[8]. In the case of free nucleon, we basically replace $R(\omega, q)$ with 1, and recover the Sullivan's convolution formula for the single nucleon.

By using (1), substantial enhancement of the sea quark distribution in nuclei was found by several authors[6, 10], and such an enhancement should be observed in the DY experiment[7]. However, recent experiment of the DY process indicates that there is no enhancement of the sea quark in nuclei[11]. Also, the sea quark enhancement due to the pion excess is not observed in the deep inelastic scattering[12]. Recently, Bertsch *et al.*[13] and Brown *et al.*[14] studied such a puzzle 'why the nuclear pion enhancement is not observed', by taking into account the modification of the gluon properties in the nucleus, or the universal scaling law due to the chiral symmetry restoration in the medium[15].

In the above discussion, the medium effect leads an excess of the number of pion in nuclei, but the pion structure function itself is assumed to be unchanged in the nuclear matter. In this letter, we concentrate on the medium effect on *the quark distribution in the pion*, and its implication for the nuclear pion enhancement. If the medium effects change the pion distribution in the nucleus, it is also important to examine whether the nuclear medium modifies the quark distribution in the pion or not. Following this scenario, we study the density dependence of the pion structure function.

The antiquark distribution of the pion appeared in (1) consists of the valence antiquark and sea quarks. For $x > 0.1^\dagger$, most contribution to the convolution formula

[†] We do not consider the region $x < 0.1$, since the nuclear shadowing effect, which we do not take into account in this letter, is important in that region.

comes from the valence antiquark part. Hence, we consider only the valence quark distribution in the pion and its medium modifications in this letter.

In the previous studies of the present author[16, 17], we have evaluated the pion structure function using the Nambu and Jona-Lasinio model (NJL) [18], and obtained reasonable agreements with experimental data. Here, we adopt the similar procedure in the nuclear medium. The NJL model, where the chiral invariance is main ingredient, describes the dynamical mass generation of the constituent quark, and reproduces $SU(3)$ meson properties successfully as a low energy effective theory of QCD[19]. It is believed that restoration of the chiral symmetry takes place at high density. Even at the nuclear matter density $\rho = \rho_0 = 0.17\text{fm}^{-3}$, the chiral symmetry may be partially restored. We will show how the pion structure function changes in the nuclear medium in terms of the NJL model.

2 Nambu and Jona-Lasinio model in medium

Before we start calculating the pion structure function in medium, we briefly give a basic procedure of the NJL model at finite density. The NJL model demonstrates the spontaneous chiral symmetry breaking of the QCD vacuum, in which the gluon degrees of freedom are assumed to be frozen into a chiral invariant 4-quark interaction. The NJL lagrangian is written as,

$$\mathcal{L}_{NJL} = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi + G_S[(\bar{\psi}t_i\psi)^2 + (\bar{\psi}i\gamma_5 t_i\psi)^2] , \quad (3)$$

where ψ denotes the quark field with current mass m , t_i are $SU(3)$ flavor matrices with the normalization $\text{tr}[t_i t_j] = \delta_{ij}/2$, and G_S the coupling constant. In this model, the constituent quark mass M and the quark condensate $\langle\bar{\psi}\psi\rangle$ have non-zero values due to the dynamical chiral symmetry breaking, if the coupling constant G_S is larger than a certain critical value. As a result, the zero mass Goldstone pion appears in the chiral limit $m = 0$ [18]. Using the realistic current quark masses, $m_u \sim 5\text{MeV}$ and $m_s \sim 150\text{MeV}$, $SU(3)$ meson properties are well reproduced in the generalized NJL model with the $U(1)_A$ anomaly[19].

For the application at finite density, we adopt the quark propagator;

$$S(p) = (\not{p} + M) \left[\frac{1}{p^2 - M^2 + i\varepsilon} + \frac{i\pi}{E} \delta(p^0 - E) \theta(k_F - |\vec{p}|) \right] \quad (4)$$

where $E = \sqrt{p^2 + M^2}$. The quark Fermi momentum k_F is related with the baryon number density. Restricting the 2-flavor, the baryon number density ρ is given by,

$$\rho = 4 \int_0^{k_F} \frac{d^3k}{(2\pi)^3}.$$

The quark self-energy is modified by the presence of other quarks in medium. From (4), the quark condensate becomes,

$$\begin{aligned} \langle \bar{\psi}\psi \rangle &= -i \text{tr} \int \frac{d^4p}{(2\pi)^4} S(p) \\ &= -\frac{N_c}{\pi^2} \int_{k_F}^{\Lambda} dp p^2 \frac{M}{\sqrt{p^2 + M^2}}. \end{aligned} \quad (5)$$

Then, the Gap equation depends on the Fermi momentum.

$$M = m - G_S \langle \bar{\psi}\psi \rangle \quad (6)$$

Apparently, the constituent quark mass M vanishes in the limit $k_F \rightarrow \infty$. In this letter, we use the 3-dimensional sharp cutoff scheme. Choice of the 3-dim. sharp cutoff makes it possible to maintain the gauge invariance in the following calculations[19].

The pion properties are also obtained by solving the Bethe-Salpeter (BS) equation. The quark-antiquark scattering matrix in the pion channel with the total momentum p is evaluated as,

$$\mathcal{T}(p) = \frac{2G_S}{1 - 2G_S\Pi(p^2)} \quad (7)$$

where

$$\Pi(p^2) = i \text{tr} \int \frac{d^4k}{(2\pi)^4} [i\gamma_5 S(k) i\gamma_5 S(k+p)]$$

The pion mass is determined by the pole of denominator at $p^2 = m_\pi^2$;

$$1 - 2G_S\Pi(p^2 = m_\pi^2) = 0$$

Straightforward calculation yields,

$$\frac{m}{M} + 2G_S p^2 I_2(p^2) = 0 \quad , \quad (8)$$

where

$$I_2(p^2) = -\frac{N_c}{4\pi^2} \int_{4(M^2+k_F^2)}^{4(M^2+\Lambda^2)} \frac{1}{\kappa^2 - p^2} \sqrt{1 - \frac{4M^2}{\kappa^2}} \quad . \quad (9)$$

Form the scattering matrix (7), the pion-quark-quark coupling constant is expressed as,

$$g_{\pi qq}^2 = \frac{1}{-\frac{d}{dp^2}[p^2 I_2(p^2)]} \quad . \quad (10)$$

We use the parameter set, $m = 5.8\text{MeV}$, $\Lambda = 615\text{MeV}$, $G_S \Lambda^2 = 9.08$, which are chosen to reproduce the pion mass 140MeV and the decay constant $f_\pi = 93.3\text{MeV}$ at zero density. Results for the constituent quark mass and the quark condensate are $M = 358\text{MeV}$ and $\langle \bar{\psi}\psi \rangle^{1/3} = -245\text{MeV}$, which are consistent with phenomenological values. All the results are basically same as those obtained in previous studies[19].

In the nuclear medium, the quark condensate decreases as the baryon number density increases, and hence the constituent mass and the pion decay constant also decrease. At the normal nuclear matter density, $\rho_0 = 0.17\text{fm}^{-3}$, values of these quantities become about 70% of their zero density values. The pion mass is almost independent of the density before the chiral phase transition. After the chiral symmetry is restored, the pion mass grows rapidly. Such a behavior is general tendency of the NJL model calculations[20].

3 Calculation of the moments of the structure function

We shall consider the medium modification of the pion structure function using the NJL model. In order to calculate the pion structure function, we adopt a formalism based on the operator product expansion (OPE) to separate the hard and soft parts of the matrix elements explicitly[21]. By doing so, it is easy to deal with the effects of finite density on the matrix elements.

As a result of the OPE, the forward scattering amplitude is written by the hard part (Wilson coefficients) and the matrix elements of local operators as illustrated in Fig.2. The twist-2 operators of the un-polarized structure function are defined as,

$$O_{\mu_1\mu_2\cdots\mu_n} = \frac{i^n}{n!} \bar{\psi} \gamma_{\mu_1} D_{\mu_2} \cdots D_{\mu_n} \psi + \text{permutations} \quad (11)$$

where D_μ is the covariant derivative of QCD. Here, we write the quark part only. Also define the matrix element A_n ,

$$\langle p | O_{\mu_1\mu_2\cdots\mu_n} | p \rangle = A_n p_{\mu_1} p_{\mu_2} \cdots p_{\mu_n} + \text{trace terms} \quad (12)$$

where $|p\rangle$ is the pion state with the momentum p . We also write the momentum of the virtual photon q , $\nu = p \cdot q$, and the Bjorken- x ; $x = -q^2/2m_\pi\nu$. This matrix element is related with the moment of the structure function;

$$\int_0^1 dx x^{n-2} F_2(x, q^2) = A_n(\log[q^2]) \quad (13)$$

where the structure function $F_2(x) = \sum e_i^2 x q_i(x)$ with the $q_i(x)$ being the quark longitudinal momentum distribution function in the pion.

Here, we choose the axial gauge $q \cdot A_\mu = 0$. Hence, multiplication of $q_{\mu_1} \cdots q_{\mu_n}$ for both sides yields,

$$A_n \nu^n = \langle p | \bar{\psi} \not{q} (q \cdot \partial)^{n-1} \psi | p \rangle \quad (14)$$

Then, we can get expressions for the n -th moment of the structure function.

We shall evaluate the matrix element of local operators in terms of the NJL model. The diagram Fig.2 indicates only the valence quark contribution up to the leading order. Hereafter, we stress on the n -th moment of the valence quark A_n^{val} , which is related with the valence quark distribution as,

$$A_n^{val} = \int_0^1 dx x^{n-1} q(x). \quad (15)$$

Throughout the following calculations, we omit the overall charge (isospin) factor for simplicity. The valence antiquark also has a similar contribution.

A_1^{val} is written by,

$$A_1^{val} p_\mu = g_{\pi qq}^2 \text{tr} \int \frac{d^4 k}{(2\pi)^4} \frac{[i\gamma_5(\not{k} + \not{p} + M)\gamma_\mu(\not{k} + \not{p} + M)i\gamma_5(\not{k} + M)]}{(k^2 - M^2 + i\varepsilon)[(k + p)^2 - M^2 + i\varepsilon]^2} \quad (16)$$

After some algebra, we find

$$\begin{aligned} A_1^{val} p_\mu &= -g_{\pi qq}^2 p_\mu [p^2 \frac{d}{dp^2} I_2(p^2) + I_2(p^2)] \\ &= -p_\mu g_{\pi qq}^2 \frac{d}{dp^2} [p^2 I_2(p^2)] \\ &= p_\mu . \end{aligned} \quad (17)$$

where we have used eq. (10) for $g_{\pi qq}$ in the last equality. Thus, we get $A_1^{val} = 1$, which manifests the Adler sum rule for the valence quark distribution function. Antiquark contribution also gives the same result.

We next consider the second moment which gives the momentum fraction carried by quarks. The 2-nd moment is defined by,

$$A_2^{val} p_\mu p_\nu = \frac{1}{2!} [\langle p | \bar{\psi} \gamma_\mu (k + p)_\nu \psi | p \rangle + \text{permutation}] , \quad (18)$$

Similar manipulation with the case of the 1-st moment gives,

$$\begin{aligned} \sim \int \frac{d^4 k}{(2\pi)^4} & \left[\frac{k_\mu k_\nu + k_\mu p_\nu}{[(k + p)^2 - M^2 + i\varepsilon]^2} + \frac{p_\mu (k + p)_\nu}{[(k + p)^2 - M^2 + i\varepsilon]^2} \right. \\ & + p_\mu \frac{d}{dp^2} \frac{p^2 k_\nu}{(k^2 - M^2 + i\varepsilon)((k + p)^2 - M^2 + i\varepsilon)} \\ & \left. + p_\mu p_\nu \frac{d}{dp^2} \frac{p^2}{(k^2 - M^2 + i\varepsilon)((k + p)^2 - M^2 + i\varepsilon)} \right] \end{aligned}$$

The first and second term give the trace terms, which are proportional to $g_{\mu\nu}$, and are irreverent in the Bjorken limit[21]. Sum of the third and forth terms becomes,

$$\begin{aligned} A_2^{val} p_\mu p_\nu &= -p_\mu p_\nu g_{\pi qq}^2 \frac{d}{dp^2} p^2 I_2 \\ &\quad - p_\mu g_{\pi qq}^2 \frac{d}{dp^2} [p^2 \int \frac{d^4 k}{(2\pi)^4} \frac{k_\nu}{(k^2 - M^2 + i\varepsilon)[(k + p)^2 - M^2 + i\varepsilon]}] \\ &= -p_\mu p_\nu g_{\pi qq}^2 \frac{d}{dp^2} [p^2 (I_2 - \frac{1}{2} I_2)] \\ &= \frac{1}{2} p_\mu p_\nu . \end{aligned} \quad (19)$$

Again, the calculation for the antiquark part gives 1/2. This is just the momentum sum rule.

$$\int_0^1 dx x [q(x) + \bar{q}(x)] = 1$$

Above result shows that the momentum sum rule is strictly satisfied in the NJL model, *i.e.* all the momentum in the pion is carried by the valence quarks at the low energy model scale. Of course, if we consider sea quark degrees of freedom, which are identified with higher order loop corrections in the NJL model[17, 22], the sea quarks carry the certain amount of the momentum fraction even at the model scale.

In the previous studies[16, 17], we had adopted the covariant parton formalism and artificial cutoff procedure, which breaks the gauge invariance. The correct treatment described here maintains the momentum sum rule.

The results for the 1-st and 2-nd moments are quite reasonable. Characteristic feature of the non-perturbative model may be seen in higher moments. It is worth mentioning that the 1-st and 2-nd moments are independent of the density.

Calculations for the higher moments require more tedious manipulations. The 3-rd moment is defined by,

$$A_3^{val} p_\mu p_\nu p_\gamma = \frac{1}{3!} [\langle p | \bar{\psi} \gamma_\mu (k+p)_\nu (k+p)_\gamma \psi | p \rangle + \text{permutation}] . \quad (20)$$

We show only the result.

$$A_3^{val} = \frac{1}{4} + g_{\pi qq}^2 \frac{N_c}{8\pi^2} I_3(p^2, M) \quad (21)$$

where

$$I_3(p^2, M) = \int_{4(M^2+k_F^2)}^{4(M^2+\Lambda^2)} d\kappa^2 \sqrt{\frac{\kappa^2 - 4M^2}{p^2}} \times \left[-\frac{1}{8p^2} \log \left| \frac{\sqrt{p^2} + \sqrt{\kappa^2}}{-\sqrt{p^2} + \sqrt{\kappa^2}} \right| + \frac{1}{4} \frac{1}{\kappa^2 - p^2} \sqrt{\frac{\kappa^2}{p^2}} \right] \quad (22)$$

Note that the 3-rd moment explicitly depends on the NJL model parameters. Whenever the density increases, the 1-st and 2-nd moments are unchanged, *i.e.* they are just numerical numbers. However, the 3-rd moment shows density dependence through

the constituent quark mass, coupling constant, and pion mass. The restoration of the chiral symmetry drives the reduction of the 3-rd moment, which will be shown later.

Similarly, the 4-th moment is calculated as,

$$A_4^{val} = \frac{1}{8} + g_{\pi qq}^2 \frac{3N_c}{16\pi^2} I_3(p^2, M) . \quad (23)$$

Higher moments are calculable in the same manner.

It is interesting to note that numerical numbers appeared in eqs. (21) and (23), $1/4$ and $1/8$, are the same as results of the delta function quark distribution $q(x) = \delta(x - 1/2)$. Such expressions are desirable. Remaining model dependent parts are corrections due to the non-perturbative bound state structure. In the limit of high density $\rho \rightarrow \infty$, the correction terms go to zero, and the structure function approaches the non-relativistic (non-interacting) limit $q(x) = \delta(x - 1/2)$.

4 Numerical results and discussions

Using the moments obtained above and performing the inverse Mellin transformation (or trial and error), we arrive at an expression of the quark momentum distribution function $q(x)$. Practically, it needs calculations up to the 4 \sim 6-th moments to determine a shape of the structure function by using eq. (15). The calculated distribution function is defined at the low energy scale, where the effective theory is supposed to work. To compare with experiments, we carry out the Q^2 evolution of the n -th moments by using the perturbative QCD[21]. The result of the effective theory plays a boundary condition for the QCD evolution. Here, we choose a low energy scale $\mu = 0.35\text{GeV}$, which is a value of the constituent quark mass, and the QCD scale parameter $\Lambda_{QCD} = 0.25\text{GeV}$. It is difficult to estimate ambiguities of the QCD evolution from such a low momentum scale. It needs more detailed studies to clarify use of the perturbative QCD below 1 GeV.

We show in Fig.3 the valence quark distribution function at $8\text{GeV}^{2\dagger}$. The solid

[†]Of course, the valence antiquark distribution has the same shape.

curve denotes the model calculation at zero density; $\rho = 0$, which is to be compared with the experimental fit indicated by the dashed curve[23]. The result is in a reasonable agreement with the experiment.

We next show the results at the nuclear matter density $\rho = \rho_0$ in Fig.4 and Fig.5. To deal with the medium effect, we use two different methods.

SET1 : Simple application of the NJL model at finite density.

SET2 : Brown-Rho scaling relation

To calculate results of SET1, we simply use the NJL model at finite density, described previously. Density dependence of the various properties is calculated by solving eqs. (6) and (7). As the matter density increases, the quark condensate, constituent quark mass, and pion decay constant decrease about 30%, whereas the pion mass and the pion-quark-quark coupling constant are almost unchanged. Such a behavior was already studied by several groups, and details are found in ref. [20].

However, the confinement of quarks are absent in the NJL model, and so this model at finite density implicitly assumes the ‘deconfined’ quark matter in the nucleus. Hence, the result obtained by the NJL model at finite density is not so reliable. Here, we adopt another possible method to incorporate the density dependence, namely, universal scaling law proposed by Brown and Rho[15]. Thus, we assume the following scaling relation;

$$\frac{M^*}{M} = \frac{g_{\pi qq}^*}{g_{\pi qq}} = \frac{f_\pi^*}{f_\pi} , \quad (24)$$

where the asterisk denotes the values of quantities in the medium. In this case, the pion mass is assumed to be constant. We use $M^*/M = 0.8$ at the normal nuclear matter density.

Results for the quark distribution function at the normal nuclear matter density are shown in Fig.4. Ratio of the quark distribution in the medium to one in the vacuum is also shown in Fig.5. As for SET1 indicated by the solid curve, density dependence of the quark distribution is weak. The calculated distribution function is almost same as one of the free pion shown in Fig.3. Only around $x \sim 1$, the

quark distribution function decreases about 10%. Indeed, values of the 3-rd and 4-th moments change only a few %. Such a weak density dependence is easily understood in the chiral limit. If we take the zero current mass $m = 0$, the resulting pion mass becomes zero $m_\pi^2 = p^2 = 0$ due to the Goldstone boson nature. Inserting $p^2 = 0$ into (10) and (22), functions $g_{\pi qq}$ and I_3 are rewritten as,

$$g_{\pi qq}^{-2}(p^2 = 0) = -\frac{d}{dp^2}[p^2 I_2(p^2)] \rightarrow \frac{N_c}{4\pi^2} \int_{4(M^2+k_F^2)}^{4(M^2+\Lambda^2)} \frac{1}{\kappa^2} \sqrt{1 - \frac{4M^2}{\kappa^2}} \ , \quad (25)$$

$$I_3(p^2 = 0) \rightarrow \frac{1}{6} \int_{4(M^2+k_F^2)}^{4(M^2+\Lambda^2)} \frac{1}{\kappa^2} \sqrt{1 - \frac{4M^2}{\kappa^2}} \ . \quad (26)$$

Namely, $g_{\pi qq}^{-2}$ and I_3 are reduced to the same function in the chiral limit. Using this result with eqs. (21) and (23), we find the 3-rd and 4-th moments become just numerical numbers,

$$A_3^{val} = \frac{1}{4} + \frac{1}{12} = \frac{1}{3} \quad (27)$$

$$A_4^{val} = \frac{1}{8} + \frac{1}{8} = \frac{1}{4} \ , \quad (28)$$

and thus density dependence of both moments vanishes. Therefore, the valence quark distribution function in the chiral limit (zero mass pion) is independent of the matter density. In addition, from the results (17, 19, 27, 28), we can write an analytical expression of the quark distribution for the zero mass pion, $xq(x) = x$ ($0 < x < 1$) at the model scale,

In the case of SET2, the medium effect on the distribution function is drastic as clearly seen in Fig.4 and 5. For $x > 0.6$, the result at the nuclear matter density shows a substantial reduction. In this case, a value of the 3-rd moment decreases about 10%, and 20% for the 4-th moment.

Finally, we discuss a connection between the nuclear pion enhancement of the sea quark distribution and the medium modification of the pion structure function. From eq. (2) calculated by the random phase approximation, it can be shown that the pion momentum distribution in the nucleus $f_{\pi NN}^A(y)$ is peaked at small y ; $y < 0.3$ [6, 10].

Thus, the integral of eq. (1) is dominated by the small y contribution. Let us recall that argument of $\bar{q}_\pi(x_\pi = x/y)$ is proportional to $1/y$. Hence, the integral (1) is sensitive to the large x_π behavior of the quark distribution function[24]. We have shown that the quark distribution function at the large x_π decreases as the density increases. Hence, we expect that the reduction of the pion structure function at the large Bjorken- x in the medium compensates the enhancement of the sea quark distribution due to the pion excess in the nucleus. In fact, it can be shown that, for $x > 0.1$, the enhancement of the sea in the nucleus almost disappears by using the density dependent pion structure function of SET2. On the other hand, the nuclear pion enhancement of the sea quark still remains in the case of SET1. More detailed discussions will be given in the forthcoming paper[24].

In conclusion, we have studied the pion structure function in the nuclear medium within the NJL model as an effective theory of QCD. We have calculated the moments of the pion structure function based on OPE to separate the soft part of the matrix elements and to maintain the gauge invariance. The result for the free pion shows a reasonable agreement with the experiment. We have shown that the quark distribution at the large x region decreases in the nuclear medium $\rho = \rho_0$. Magnitude of the modification depends on treatments of the medium effect. We have found that the medium modification of the pion structure function is small within the NJL model around the nuclear matter density. Instead, if we use the Brown-Rho scaling relation, the quark distribution function shows substantial reduction for the large x . We note that the quark distribution function in the high density limit approaches the non-relativistic one, $q(x) = \delta(x - 1/2)$ (at the model scale). It is also interesting to study the medium modification of other meson structure functions, which is under consideration[24].

We have also discussed the nuclear pion enhancement of the sea quarks in the nucleus in terms of the density dependent pion structure function. The reduction of the pion structure function at the large x may lead a suppression of the sea quark distribution due to the pion cloud. Such a tendency seems to be consistent with

the available experiments, where the sea quark enhancement in the nucleus is not observed.

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Figure Captions

Fig. 1

The nucleon-hole and delta-hole excitations contribute to the deep inelastic lepton nucleus scattering. Solid line denotes the nucleon, and the double one the delta. The pion and virtual photon are depicted by the dashed and wavy curves, respectively.

Fig. 2

Graphical representation of the operator product expansion. The quark and the pion are depicted by the solid and dashed curves, respectively. Wavy curve denotes the virtual photon. Left hand diagram shows the forward scattering amplitude of the pion in the NJL model.

Fig. 3

The valence quark momentum distribution function at $Q^2 = 8\text{GeV}^2$ as a function of the Bjorken x . The solid curve denotes theoretical calculation for the free pion ($\rho = 0$), and the dashed curve the experimental fit[23].

Fig. 4

The valence quark distribution function of the pion in the nuclear medium $\rho = \rho_0$ at $Q^2 = 8\text{GeV}^2$. The result of SET1 is depicted by the solid curve, and SET2 by the dashed curve. See text for detail.

Fig. 5

Ratio of the quark distribution functions at the normal nuclear matter density ($\rho = \rho_0$) to one of the free pion ($\rho = 0$); $q(x)_{\rho=\rho_0}/q(x)_{\rho=0}$. Results of SET1 and SET2 are depicted by solid and dashed curves, respectively.

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